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PROBLEMS

Problem. Given a partially ordered set P, what subsets of P are images of order-preserving idempotent functions $f: P \longrightarrow P$? (If P is a complete lattice, the answer is: any subset $A \subseteq P$ which is a complete lattice in the induced order.)

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Problems belong to the folklore

1. What are lattices of congruence relations of groupoids (algebras of finite type)? (conjecture: all algebraic lattices)
2. What is the concrete structure of the set of congruence relations for an algebra?

Other problems

1. (See 1. above) Given a complete lattice L, what is the minimum number of operations required to represent L as the congruence lattice of an infinitary algebra? What is the minimum number of operations of rank less than the cordinality of L?
2. If L is a complete (resp., algebraic modular lattice and G is a group, is it always possible to find some infinitary (resp., finitary) algebra A s.t.

   (i) $G \cong \mathrm{Aut}(A)$

   (ii) $L \cong \mathrm{Con}(A)$

   (iii) in $\mathrm{Con}(A)$ $\Theta \vee \Phi = \Theta \Phi \Theta$ for any Θ, Φ?

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1. Problem: Let $V_c(K)$ denote the variety of lattices generated by the congruence lattices of algebras in K, where K is a variety. If $V_c(K)$ is proper, must it be included in the variety of modular lattices?

2. PROBLEM: J. B. Nation has shown (1972) that not every variety of lattices is of the form $V_c(K)$. Characterize $\{V_c(K):K \text{ a variety}\}$. Is this class a sublattice of the lattice of lattice varieties?

3. PROBLEM: If K is a congruence-modular variety of algebras of finite type, must every finite member of K have a finitely based equational theory?

4. PROBLEM: Are finitely generated free lattices weakly atomic?

5. PROBLEM: If $V(A) \prec V_1$ in the lattice of lattice varieties, and A is a finite lattice, must $V_1 = V(B)$ for some finite B? ($\prec$ means "covered", $V(A)$ is the variety generated by A.)

6. PROBLEM: Is the set of "universal" first order sentences true in the free lattice FL_3 a recursive set?

I make no claim to having originated these problems.

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If L is a uniquely complemented lattice satisfying a proper lattice identity then L is distributive. (Classically known for modular identity)

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1. Let G be group of automorphisms of a totally ordered set L. Does there exist an integer n such that for every L if G is m-transitive then for every $K \geqslant n$ G is K-transitive.

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PROBLEMS ON COMPACT SEMILATTICES

1. Let S be a compact, metric, one-dimensional semilattice. Suppose that $\varphi: S \to I$ is an open, monotone, epimorphism. Must φ be an isomorphism?

2. Let S be a compact, metric, finite-dimensional semilattice with small semilattices. Let $x, y \in S$. Does there exist a closed subsemilattice A of S such that $\dim A < \dim S$ and A separates x and y in S?

3. Let S be a compact, connected, finite-dimensional semilattice with small semilattices. Is A the strict projective limit of locally connected semilattices?

4. Let S be a compact, connected, locally connected, one-dimensional semilattice. Is S the strict projective limit of one-dimensional polyhedral semilattices?

5. Let A be a compact space with a closed partial order. Is there a continuous isotone map of A into a compact semilattice S where $\dim A = \dim S$?

6. Consider the class $\mathcal{C}$ of semilattices generated by the min interval by the operations of forming finite products, quotients and closed subsemilattices. Is the class $\mathcal{C}$ precisely the class of compact topological semilattices of finite breadth?

7. Do compact semilattices of finite breadth have the congruence extension property? Does the class $\mathcal{C}$?

8. Let U be an open cover of a compact semilattice S with small semilattices. Does there exist a closed congruence ρ such that the congruence classes of ρ refine U and S/ρ is locally connected and finite-dimensional.

9. Let S be a topological semilattice on an n-cell with boundary B such that $0 \notin B$. Is $B^2 = S$? If $x \in B$ does there exist $y \in B$ such that $xy = 0$?

10. Let S be a one-dimensional, compact, connected similattice with a closed set of end points. Does S have small semilattices?

11. Let S be a topological semilattice on a Peano continuum. Is S an AR?

12. Let S be an n-dimensional semilattice on a Peano continuum. Does S contain an n-cell?

13. Let S be a compact, connected, n-dimensional semilattice. Does there exist $x \in S$ such that $\dim xS = n$?

14. Let S be a locally compact, connected, locally connected semilattice. Is S arcwise connected? Is it acyclic? Suppose that S is not locally connected?

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