

ABSOLUTELY SUMMING OPERATORS ON BESOV SPACES

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ABSTRACT. Let I_n be the identity operator on an n -dimensional Banach space E_n . In this paper we establish upper estimates, in terms of the cotype of E_n , for some ideal norms of I_n . This results are applied to study absolutely summing operators on Besov spaces.

1. INTRODUCTION

The class of all (bounded linear) operators between arbitrary Banach spaces is denoted by \mathcal{L} , while $\mathcal{L}(E, F)$ stands for the space of those operators acting from E into F , and which is equipped with the usual *operator norm*

$$\|S\| = \|S : E \rightarrow F\| := \sup \{\|Sx\| : \|x\| \leq 1\}.$$

The set $\mathcal{F}_n(E, F)$ consists of all $S \in \mathcal{L}(E, F)$ such that $S(E) := \{Sx : x \in E\}$ is at most n -dimensional. The dual of E is denoted by E' .

We refer to [7] for definitions and well-known facts of the operator ideals $[\mathcal{M}_{r,s}, \mu_{r,s}]$, $[\Pi_{p,q}, \pi_{p,q}]$ and $[\mathcal{I}_r, i_r]$ of (r, s) -mixing, absolutely (p, q) -summing and r -integral operators, respectively. For $p = q$ we have the operator ideal $[\Pi_p, \pi_p]$ of absolutely p -summing operators (for $p = 1$ we simply speak of an absolutely summing operator). We shall freely make use of the results given there, omitting specific references.

Let us recall that a Banach space E is said to be of (Rademacher) *cotype* q , with $2 \leq q < \infty$, if there exists a constant k such that

$$\left(\sum_{i=1}^n \|x_i\|^q \right)^{1/q} \leq k \int_0^1 \left\| \sum_{i=1}^n r_i(t)x_i \right\| dt$$

for all finite families of elements $x_1, \dots, x_n \in E$, where r_i denotes the i -th Rademacher function. We put $K_q(E) := \inf k$.

If $1 \leq p \leq \infty$, then the dual exponent p' is defined by $1/p + 1/p' = 1$.

By c, c_1, c_q, \dots , we always denote positive constants, eventually depending on certain exponents, but not on other quantities like operators, Banach spaces, or natural numbers.

2. COTYPE AND UPPER BOUNDS FOR SOME IDEAL NORMS

In this section, I_n denotes the identity operator on an n -dimensional Banach space E_n . We start our considerations with the

Lemma 2.1. *Let $1 \leq s \leq 2$ and $n, m = 1, 2, \dots$. Then for every operator $T : l_\infty^m \rightarrow E_n$ one has:*

$$(i) \pi_s(T) \leq c_q K_q(E_n) n^{1/s-1/q} \|T\|, \text{ where } 2 < q.$$

$$(ii) \pi_s(T) \leq c K_2(E_n) [1 + \log K_2(E_n)]^{1/2} n^{1/s-1/2} \|T\|.$$

PROOF. (i) We have $\pi_{q,1}(I_n) \leq K_q(E_n)$. From [9, p. 150] and [3] (see also [9, p. 160]) we obtain

$$\begin{aligned} \pi_2(T) &\leq c n^{1/2-1/q} \pi_{q,2}(T) \\ &\leq c_q n^{1/2-1/q} \pi_{q,1}(T) \\ &\leq c_q K_q(E_n) n^{1/2-1/q} \|T\|. \end{aligned}$$

Using the multiplication formula

$$[\mathcal{I}_2, i_2] \circ [\Pi_2, \pi_2] \subseteq [\mathcal{I}_1, i_1]$$

and the fact that $i_2(I_n) = n^{1/2}$, we also have

$$i_1(T) \leq c_q K_q(E_n) n^{1/q'} \|T\|.$$

Given $1 < s < 2$, we define θ by $1/s = (1 - \theta) + \theta/2$. Combining the above inequalities we arrive at

$$\begin{aligned} \pi_s(T) &\leq \pi_1(T)^{1-\theta} \pi_2(T)^\theta \\ &\leq c_q K_q(E_n) n^{1/s-1/q} \|T\|, \end{aligned}$$

concluding the proof of (i).

(ii) A result of Maurey [4] (see also [9, p. 69]) states

$$\pi_2(T) \leq c K_2(E_n) [1 + \log K_2(E_n)]^{1/2} \|T\|.$$

The result follows in a similar way to the proof of part (i), using now the above inequality. \square

We also have the

Lemma 2.2. *Let $2 < q, 2 \leq s \leq q$ and $n, m = 1, 2, \dots$. Then for every operator $T : l_\infty^m \rightarrow E_n$ one has*

$$\pi_{s,1}(T) \leq c K_q(E_n) n^{1/s-1/q} \|T\|.$$

PROOF. As in the proof of Lemma 2.1(i) we have

$$\pi_{q,2}(T) \leq c_1 \pi_{q,1}(T) \leq c_1 K_q(E_n) \|T\|,$$

and from [9, p. 150] we get

$$\pi_{s,1}(T) \leq \pi_{s,2}(T) \leq c_2 n^{1/s-1/q} \pi_{q,2}(T).$$

From this estimates we obtain the result. □

The next inequalities are the key for our later work.

Proposition 2.3. *Let $1 \leq s \leq 2$ and $n = 1, 2, \dots$. Then*

- (i) $\mu_{s',1}(I_n) \leq c_q K_q(E_n) n^{1/s-1/q}$, where $2 < q$.
- (ii) $\mu_{s',1}(I_n) \leq c K_2(E_n) [1 + \log K_2(E_n)]^{1/2} n^{1/s-1/2}$.

PROOF. (i) Given $x_1, \dots, x_m \in E_n$ we define $X : l_\infty^m \rightarrow E_n$ by

$$Xy := \sum_{i=1}^m \eta_i x_i \quad \text{for } y = (\eta_1, \dots, \eta_m) \in l_\infty^m.$$

Note that

$$\|X\| \leq \sup \left\{ \sum_{i=1}^m |\langle x_i, a \rangle| : \|a\| \leq 1 \right\}.$$

Let S be a linear map from E_n into a Banach space G . Then it follows from Lemma 2.1(i) that

$$\begin{aligned} \pi_1(SX : l_\infty^m \rightarrow G) &\leq \pi_s(X : l_\infty^m \rightarrow E_n) \pi_{s'}(S : E_n \rightarrow G) \\ &\leq c_q K_q(E_n) n^{1/s-1/q} \|X : l_\infty^m \rightarrow E_n\| \pi_{s'}(S : E_n \rightarrow G). \end{aligned}$$

Hence, if (e_i) denotes the standard basis of l_∞^m , we have

$$\begin{aligned} \sum_{i=1}^m \|Sx_i\| &= \sum_{i=1}^m \|SXe_i\| \leq \pi_1(SX : l_\infty^m \rightarrow G) \\ &\leq c_q K_q(E_n) n^{1/s-1/q} \|X : l_\infty^m \rightarrow E_n\| \pi_{s'}(S : E_n \rightarrow G) \end{aligned}$$

and this implies that

$$\pi_1(S : E_n \rightarrow G) \leq c_q K_q(E_n) n^{1/s-1/q} \pi_{s'}(S : E_n \rightarrow G).$$

This and the formula

$$[\mathcal{M}_{s',1}, \mu_{s',1}] = [\Pi_{s'}, \pi_{s'}]^{-1} \circ [\Pi_1, \pi_1]$$

yield part (i).

From Lemma 2.1(ii), one can proved similarly (ii). □

Remark. Upper estimates for $\mu_{r,s}(I_n)$, without use of cotype constants, where obtained in [1] by a different method.

Proposition 2.4. *Let $2 < q, 2 \leq s \leq q$ and $n = 1, 2, \dots$. Then*

$$\pi_{s,1}(I_n) \leq c K_q(E_n) n^{1/s-1/q}.$$

PROOF. For $m = 1, 2, \dots$, we define $X \in \mathcal{L}(l_\infty^m, E_n)$ as in Proposition 2.3. From Lemma 2.2 we obtain

$$\begin{aligned} \left(\sum_{i=1}^m \|x_i\|^s\right)^{1/s} &\leq \pi_{s,1}(X) \leq c K_q(E_n) n^{1/s-1/q} \|X\| \\ &\leq c K_q(E_n) n^{1/s-1/q} \sup \left\{ \sum_{i=1}^m |\langle x_i, a \rangle| : \|a\| \leq 1 \right\}, \end{aligned}$$

which yields the desired inequality. □

For every operator $S \in \mathcal{L}(E, F)$ the n -th approximation number is defined by

$$a_n(S) := \inf \{ \|S - L\| : L \in \mathcal{F}_{n-1}(E, F) \}.$$

We put

$$\mathcal{L}_{p,q}^{(a)} := \{ S \in \mathcal{L} : (a_n(S)) \in l_{p,q} \}$$

and

$$L_{p,q}^{(a)}(S) := \|(a_n(S))\|_{p,q} \text{ for } S \in \mathcal{L}_{p,q}^{(a)},$$

where $[l_{p,q}, \|\cdot\|_{p,q}]$, $0 < p, q \leq \infty$, stands for the quasi-normed Lorentz sequence space (cf. [8, (2.1)]) (for $p = q$ we get the classical space of p -summable sequences denoted by l_p). Then $[\mathcal{L}_{p,q}^{(a)}, L_{p,q}^{(a)}]$ becomes a quasi-normed operator ideal (see also [8, (2.3)]).

For later use we state the

Proposition 2.5. *Let E be a Banach space.*

(i) *If F is a Banach space of cotype q , with $2 < q$, and $1 \leq s \leq 2$, then*

$$\mathcal{L}_{t,1}^{(a)}(E, F) \subseteq \mathcal{M}_{s',1}(E, F),$$

where $1/t := 1/s - 1/q$.

(ii) If F is a Banach space of cotype 2 and $1 \leq s < 2$, then

$$\mathcal{L}_{r,1}^{(a)}(E, F) \subseteq \mathcal{M}_{s',1}(E, F),$$

where $1/r := 1/s - 1/2$.

(iii) If F is a Banach space of cotype q and $2 \leq s < q$, then

$$\mathcal{L}_{t,1}^{(a)}(E, F) \subseteq \mathcal{M}_{s'-\epsilon,1}(E, F),$$

where $1/t := 1/s - 1/q$ and $\epsilon > 0$.

PROOF. (i) Let $S \in \mathcal{F}_n(E, F)$. We have the factorization $S = JIS_0$:

$$S : E \xrightarrow{S_0} S(E) \xrightarrow{I} S(E) \xrightarrow{J} F,$$

where S_0 is the astriction of S , I is the identity operator on $S(E)$ and J is the natural injection. From Proposition 2.3(i) we have

$$\begin{aligned} \mu_{s',1}(S) &\leq \|J\| \mu_{s',1}(I) \|S_0\| \\ &\leq c_q K_q(S(E)) n^{1/s-1/q} \|S_0\| \\ &\leq c_q K_q(F) n^{1/t} \|S\|. \end{aligned}$$

Given $T \in \mathcal{L}_{t,1}^{(a)}(E, F)$ and using a representation theorem of Pietsch [8, (2.3.8)], we have $T = \sum_{k=0}^{\infty} T_k$ (convergence in the operator norm) with $T_k \in \mathcal{F}_{2^k}(E, F)$ and $(2^{k/t} \|T_k\|) \in l_1$. Hence

$$\sum_{k=0}^{\infty} \mu_{s',1}(T_k) \leq c_q K_q(F) \sum_{k=0}^{\infty} 2^{k/t} \|T_k\| < \infty,$$

and then $\sum_{k=0}^{\infty} T_k$ is convergent in the Banach space $\mathcal{M}_{s',1}(E, F)$. Therefore $T \in \mathcal{M}_{s',1}(E, F)$. This completes the proof of (i).

Similarly we obtain (ii) and (iii), employing now Proposition 2.3(ii) and Proposition 2.4, respectively, and using

$$\Pi_{s,1}(E, F) \subseteq \mathcal{M}_{s'-\epsilon,1}(E, F)$$

(inclusion given in [7, (20.1.12)]). □

3. ABSOLUTELY SUMMING OPERATORS BETWEEN BESOV SPACES

Let E be any Banach space. Let $-\infty < \sigma < +\infty$ and $1 \leq p, u \leq \infty$. As in [5] (cf. also [8, (5.4)]) the *Besov sequence space* $[b_{p,u}^\sigma, E]$ consists of all E -valued sequences $(x_{m,h})$ indexed by

$$\mathbb{P} := \{(m, h) : m = 0, 1, \dots ; h = 1, \dots, 2^m\}$$

such that the norm

$$\|(x_{m,h})\|_{[b_{p,u}^\sigma, E]} := \left\{ \sum_{m=0}^{\infty} \left[2^{m\sigma} \left(\sum_{h=1}^{2^m} \|x_{m,h}\|^p \right)^{1/p} \right]^u \right\}^{1/u}$$

is finite. In the cases $p = \infty$ or $u = \infty$ the usual modifications are required. If E is the scalar field, then we simply write $b_{p,u}^\sigma$.

Let $A = (\alpha_{m,h;n,k})$ be any matrix. By $a_{m,h}$ we denote the scalar sequence $(\alpha_{m,h;n,k})$ where (m, h) is fixed. We define $\mathbf{a} := (a_{m,h})$. We say that the matrix A belongs to $[b_{p,u}^\sigma, b_{q,v}^\tau]$ if this is true for the $b_{q,v}^\tau$ -valued sequence \mathbf{a} .

Putting

$$S_A : (\eta_{n,k}) \rightarrow \left(\xi_{m,h} := \sum_{n=0}^{\infty} \sum_{k=1}^{2^n} \alpha_{m,h;n,k} \eta_{n,k} \right)$$

we get an operator $S_A \in \Pi_s(b_{q',v'}^\tau, b_{p,u}^\sigma)$, where $s := \max(p, u)$ (see [5]).

In the following we deal with the natural embedding I from $b_{p,u}^\sigma$ into $b_{q,v}^\tau$, which exists if the condition $\sigma - \tau > \max(1/q - 1/p, 0)$ holds.

In order to study the cotype of $b_{p,u}^\sigma$, with $1 \leq p, u < \infty$, we consider the following facts:

Let (E_i) be a sequence of Banach spaces with $i \in \mathbb{N}$. Then $[l_p, E_i]$ denotes the Banach space of all sequences $x = (x_i)$, with $x_i \in E_i$ for $i \in \mathbb{N}$, such that

$$\|x\|_{l_p} := \left(\sum_{i=1}^{\infty} \|x_i\|^p \right)^{1/p} < \infty,$$

where $1 \leq p < \infty$. If we suppose that every space E_i is of cotype q and $\sup \{K_q(E_i) : i \in \mathbb{N}\} < \infty$, then $[l_p, E_i]$ is of cotype $\max(p, q)$. The proof of this result is given in [6] for $p \leq q$, and the case $p > q$ can be obtained in a similar form.

We apply the above consideration to the special sequence of the 2^m -dimensional Banach spaces $2^{m\sigma} l_p^{2^m}$ with $m = 0, 1, \dots$ (see [8, (B.1.4)]). Since $b_{p,u}^\sigma := [l_u, 2^{m\sigma} l_p^{2^m}]$, we easily get that the cotype of $b_{p,u}^\sigma$ is $\max(p, u, 2)$.

Now we are in a position to prove the

Theorem 3.1. *Let $-\infty < \sigma, \tau < +\infty$, $1 \leq p, u \leq \infty$ and $1 < q, v \leq \infty$. Let $s := \max(p, u)$, $t := \max(q', v', 2)$ and suppose that one of the following conditions is satisfied:*

- (a) $s \geq 2$ if $t > 2$.
- (b) $s > 2$ if $t = 2$.
- (c) $2 < s' < t$.

Let $A \in [b_{p,u}^\sigma, b_{q,v}^\tau]$ and

$$1/r := \begin{cases} 1 - 1/q - 1/p & \text{if } q' \leq p \\ 0 & \text{if } p \leq q'. \end{cases}$$

If $\sigma + \tau > 1 + 1/r - 1/s - 1/t$, then

$$R_A : (\eta_{n,k}) \rightarrow \left(\xi_{m,h} := \sum_{n=0}^{\infty} \sum_{k=1}^{2^n} \alpha_{m,h;n,k} \eta_{n,k} \right)$$

verifies $R_A \in \Pi_1(b_{p,u}^\sigma, b_{p,u}^\sigma)$.

PROOF. First we assume that the conditions (a) or (b) are verified. We have the factorization $R_A = S_A I$:

$$R_A : b_{p,u}^\sigma \xrightarrow{I} b_{q',v'}^{-\tau} \xrightarrow{S_A} b_{p,u}^\sigma$$

with $S_A \in \Pi_s(b_{q',v'}^{-\tau}, b_{p,u}^\sigma)$.

By [2] there exists a constant c such that, for all n ,

$$n^{\sigma+\tau-1/r} a_n(I) \leq c.$$

Hence $I \in \mathcal{L}_{t_0,1}^{(a)}(b_{p,u}^\sigma, b_{q',v'}^{-\tau})$ if $\sigma + \tau - 1/r > 1/t_0$, and from Proposition 2.5(i) or 2.5(ii) we obtain

$$I \in \mathcal{M}_{s,1}(b_{p,u}^\sigma, b_{q',v'}^{-\tau})$$

whenever $1/s' = 1/t_0 + 1/t$.

Using the inclusion

$$[\Pi_s, \pi_s] \circ [\mathcal{M}_{s,1}, \mu_{s,1}] \subseteq [\Pi_1, \pi_1]$$

we finally obtain $R_A \in \Pi_1(b_{p,u}^\sigma, b_{p,u}^\sigma)$ whenever

$$\sigma + \tau > 1/r + 1/s' - 1/t.$$

This concludes the desired part of the proof.

If condition (c) is satisfied, the proof follows using the same method and Proposition 2.5(iii). □

Let $\sigma > 0$ and $1 \leq p, u \leq \infty$. The Besov function space $[B_{p,u}^\sigma(0, 1), E]$ consists of certain E -valued functions defined on the unit interval $[0, 1]$ (see [8, (6.4)]). If E is the scalar field, then we simple write $B_{p,u}^\sigma(0, 1)$.

For $m > \sigma + 1 - 1/p$, the Ciesielski-Ropela transform is denoted by C_m , which establishes an isomorphism between

$$B_{p,u}^\sigma(0, 1) \quad \text{and} \quad l_p^m \oplus b_{p,u}^{\sigma-1/p+1/2}.$$

Further information is also given in [8, (6.4)].

A kernel K defined on the unit square $[0, 1] \times [0, 1]$ belongs to

$$[B_{p,u}^\sigma(0, 1), B_{q,v}^\tau(0, 1)]$$

if the function-valued function

$$K_X : \xi \rightarrow K(\xi, \cdot)$$

belongs to $[B_{p,u}^\sigma(0, 1), B_{q,v}^\tau(0, 1)]$.

In the following we deal with the embedding I_B from $B_{p,u}^\sigma(0, 1)$ into $B_{q,v}^\tau(0, 1)'$ which exists if the condition $\sigma + \tau > 1/p + 1/q - 1$ is satisfied.

Finally, we formulate the

Theorem 3.2. *Let $\sigma, \tau > 0, 1 \leq p, u \leq \infty$ and $1 < q, v < \infty$. Let $s := \max(p, u)$, $t := \max(q', v', 2)$ and suppose that one of the following conditions is satisfied:*

- (a) $s \geq 2$ if $t > 2$.
- (b) $s > 2$ if $t = 2$.
- (c) $2 < s' < t$.

Let $K \in [B_{p,u}^\sigma(0, 1), B_{q,v}^\tau(0, 1)]$ and

$$1/r := \begin{cases} 1 - 1/q - 1/p & \text{if } q' \leq p \\ 0 & \text{if } p \leq q'. \end{cases}$$

If $\sigma + \tau > 1/p + 1/q + 1/r - 1/s - 1/t$, then

$$T_K : f(\eta) \rightarrow \int_0^1 K(\xi, \eta) f(\eta) d\eta$$

verifies $T_K \in \Pi_1(B_{p,u}^\sigma(0, 1), B_{p,u}^\sigma(0, 1))$.

PROOF. First we assume that the conditions (a) or (b) are verified. Let $m > \max(\sigma + 1 - 1/p, \tau + 1 - 1/q)$. From [8, (6.4.13)] the embedding I_B is related to embedding operators I_m and I_b acting between sequence spaces by

$$\begin{array}{ccc} B_{p,u}^\sigma(0, 1) & \xrightarrow{I_B} & B_{q,v}^\tau(0, 1)' \\ C_m \downarrow & & \uparrow C'_m \\ l_p^m \oplus b_{p,u}^{\sigma-1/p+1/2} & \xrightarrow{I_m \oplus I_b} & (l_q^m)' \oplus (b_{q,v}^{\tau-1/q+1/2})'. \end{array}$$

Since $(b_{q,v}^{\tau-1/q+1/2})' = b_{q',v'}^{-\tau+1/q-1/2}$, we obtain from C'_m that $B_{q,v}^\tau(0, 1)'$ has cotype t .

By [2] we have for the embedding

$$J_b : b_{p,u}^{\sigma-1/p+1/2} \rightarrow b_{q',v'}^{-\tau+1/q-1/2}$$

that exists a constant c such that, for all n ,

$$n^{\sigma+\tau-1/p-1/q-1/r+1} a_n(J_b) \leq c.$$

Hence, from the above diagram

$$I_B \in \mathcal{L}_{t_0,1}^{(a)}(B_{p,u}^\sigma(0,1), B_{q,v}^\tau(0,1)')$$

if $\sigma + \tau - 1/p - 1/q - 1/r + 1 > 1/t_0$, and consequently applying Proposition 2.5(i) or 2.5(ii) we have

$$I_B \in \mathcal{M}_{s,1}(B_{p,u}^\sigma(0,1), B_{q,v}^\tau(0,1)')$$

whenever $1/t_0 = 1/s' - 1/t$.

The operator T_K admits the factorization $T_K = S_K I_B$:

$$T_K : B_{p,u}^\sigma(0,1) \xrightarrow{I_B} B_{q,v}^\tau(0,1)' \xrightarrow{S_K} B_{p,u}^\sigma(0,1),$$

where $S_K(a) := \langle K_X(\cdot), a \rangle$, and we know by [8, (6.4.16)] that

$$S_K \in \Pi_s(B_{q,v}^\tau(0,1)', B_{p,u}^\sigma(0,1)).$$

From the preceding considerations and the inclusion

$$[\Pi_s, \pi_s] \circ [\mathcal{M}_{s,1}, \mu_{s,1}] \subseteq [\Pi_1, \pi_1],$$

we obtain that $T_K \in \Pi_1(B_{p,u}^\sigma(0,1), B_{p,u}^\sigma(0,1))$ if $\sigma + \tau - 1/p - 1/q - 1/r + 1 > 1/s' - 1/t$. This proves the desired part of the assertion.

The proof under the condition (c) is analogous, using now Proposition 2.5(iii). □

Remarks. (1) Adaptation of the proof of Theorem 3.2 yields the corresponding result for appropriate kernels $[B_{p,u}^\sigma(0,1), L_q(0,1)]$ of Besov-Hille-Tamarkin type.

(2) We do not know whether the conditions given in Theorems 3.1 and 3.2 are necessary for the absolutely summing property of the operator.

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